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# PROPAGATION EFFECTS OF STEEP STRIATIONS

Science Applications, Inc.  
1200 Prospect Street  
La Jolla, California 92037

30 November 1976

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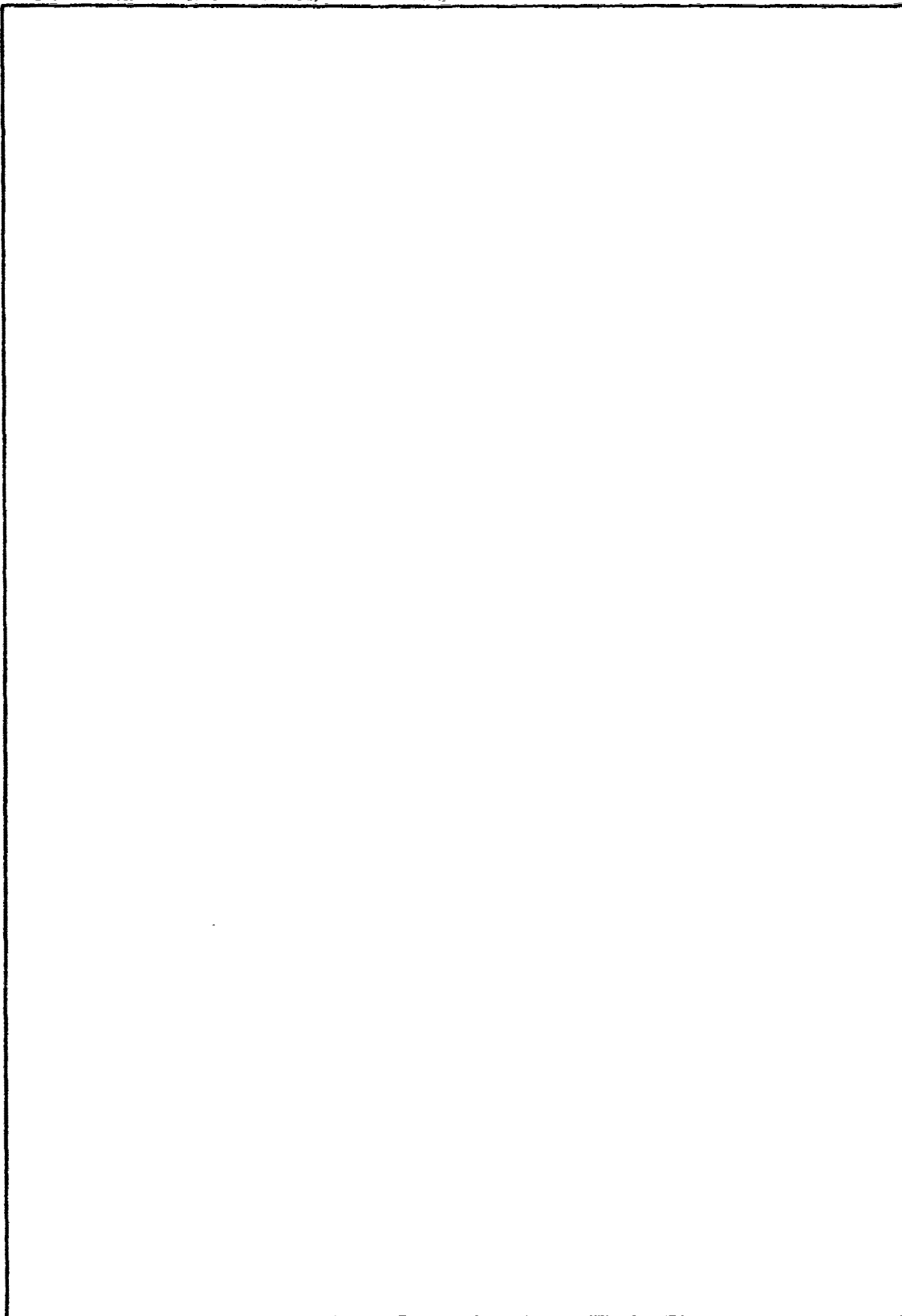
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# TABLE OF CONTENTS

|                                       | <u>Page</u> |
|---------------------------------------|-------------|
| 1. INTRODUCTION AND SUMMARY . . . . . | 3           |
| 2. ANALYTIC METHODS . . . . .         | 7           |
| 2.1 Analysis. . . . .                 | 7           |
| 2.2 Implementation. . . . .           | 21          |
| 3. RESULTS. . . . .                   | 25          |
| REFERENCES. . . . .                   | 35          |

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## ILLUSTRATIONS

|   | <u>Page</u> |
|---|-------------|
| 1. Specific and Average Power Spectra Gaussian Profile. . . . .   | 11          |
| 2. Specific and Average Power Spectra Rod Profile . . . . .   | 12          |
| 3. Average Bit Error Probability Versus rms Phase Fluctuation<br>for Noncoherent FSK Modulation and 15dB Average Signal-to-<br>Noise Ratio. . . . . | 29          |
| 4. Contours of Bit Error Probability. . . . .   | 30          |
| 5. Contours of $S_4$ ; Gaussian . . . . .   | 31          |
| 6. Contours of $S_4$ ; Rod. . . . .   | 32          |
| 7. Contours for 1% Probability of Fades > 12dB. . . . .   | 34          |

## 1. INTRODUCTION AND SUMMARY

This report describes the fluctuations in intensity that occur when a radio signal traverses striated ionization. The ionization is modeled as a collection of cylinders of electron density randomly positioned in space. This work is a continuation of a previous analysis<sup>1</sup>. The major improvements on that analysis are:

1. The use of fast Fourier transforms to calculate the signal a distance  $d$ , beyond the ionization instead of the method of rays and caustics used previously.
2. The analysis is carried out for two extreme profiles of striation electron density; the Gaussian profile previously studied and a rod profile representing an extreme steepening of the striation edge.
3. Results of systems significance are presented graphically as contours on a  $\lambda d$  versus  $\Delta\phi_{\text{rms}}$  plot where  $\lambda$  is the signal wavelength and  $\Delta\phi_{\text{rms}}$  is the root-mean-square signal phase variation directly produced by the ionization.

The last improvement allows the difference between the extreme profiles' effects to be easily seen as a function of the parameters  $\lambda d$  and  $\Delta\phi_{\text{rms}}$ .

The set of striations used is the same as that used previously<sup>1</sup>, a set of 210 striations set randomly in an area 64 km wide by a thickness  $T$ . The thickness does not enter the calculations since we use the thin-screen approximation. This approximation, whose validity criteria were previously derived<sup>1</sup> for Gaussian profiles, is adequate to demonstrate the propagation effects for the striations considered in this report. We do not foresee any qualitative changes by its removal and intend to demonstrate this in future works.

We find the rod and Gaussian profiles to produce similar propagation effects for  $\lambda d > 3 \text{ km}^2$  where the dependence on  $\lambda d$  is weakening. In the high  $\lambda d$  regime, amplitude scintillation onsets at  $\Delta\phi_{\text{rms}} \approx 0.3$  radians. As  $\lambda d$  becomes smaller the rod and Gaussian profile effects diverge with onset for Gaussian effects occurring at

$$\Delta\phi_{\text{rms}} = \frac{1}{\lambda d}$$

whereas

$$\Delta\phi_{\text{rms}} = \frac{1}{\sqrt{3\lambda d}}$$

for the onset of rod effects.

Thus the profile shape is relatively unimportant at UHF ( $\lambda \sim 10^{-3} \text{ km}$ ), but becomes very important at K Band ( $\lambda \sim 10^{-5} \text{ km}$ ). We assume  $10^2 < d < 10^4 \text{ km}$  for typical situations. For  $d = 10^3$  and K Band the Gaussian profile effects onset

$$\text{at } \Delta\phi_{\text{rms}} = 100 \text{ radians}$$

while the rod profile effects onset

$$\text{at } \Delta\phi_{\text{rms}} = 6 \text{ radians.}$$

The rms phase is given by

$$\Delta\phi_{\text{rms}} = 2.818 \cdot 10^{-3} \lambda n_0 Q_{\text{rms}}$$

where  $n_0$  is the striation on-axis electron density ( $\text{cm}^{-3}$ ) and  $Q$  is the normalized

integrated electron density. For the particular set of striations used,  $Q_{rms} = 2.07$  km for the Gaussian profile and  $Q_{rms} = 2.55$  km for the rod profile. This indicates  $\Delta\phi_{rms}$  to be relatively insensitive to profile shape for a given set of striations and values of  $n_0$  and  $\lambda$ . However, the value of  $\Delta\phi_{rms}$  required for the onset of degradation is highly sensitive to profile when  $\lambda d < 3 \text{ km}^2$ . The set of striations we have used have sizes ranging from 323m to 1614m with a maximum number at 775m. One can scale our results to different sizes by changing each size  $a$  to  $\delta a$  and scaling  $\lambda z$  as  $\delta^{-2}$ . The size distribution used is a discrete approximation to the continuous Chesnut distribution which gives the analytic formula for Gaussian profiles

$$Q_{rms}^2 = \frac{\sqrt{8}}{3} \pi a_0^3 \left(\frac{M}{L}\right)$$

where  $a_0$  is a characteristic size of the distribution ( $\sim 0.76$  km for the set used),  $M$  is the total number of striations imbedded in an area  $L \times T$ . Since  $T$  does not matter, if we scale  $a_0$  and  $L$  to  $\delta a_0$  and  $\delta L$ ,  $Q_{rms}$  scales as  $\delta$ .

This study has contrasted the propagation effects of smooth versus abrupt profiles for a set of striations. The abrupt profiles produce more severe propagation effects than smooth profiles at high frequencies. Future work will address this question for other sets of striation sizes. We do not foresee other sets of striations producing effects more severe than were found here for the abrupt profiles. We illustrate this statement with an example. Suppose we are at the threshold of rod effects at  $\lambda d = 0.1$  and  $\Delta\phi_{rms} = \sqrt{10}$ . If we change the sizes of all striations by dividing them and the width  $L$  by  $\sqrt{10}$  we effectively have changed  $\lambda d$  to 1 and  $\Delta\phi_{rms}$  to 1 which keeps us at the threshold of rod-effects (Gaussian effects get worse). If we continued to

divide the sizes we would reach  $\Delta\phi_{\text{rms}} < 0.3$  and propagation effects would cease. We believe the results presented in this report for rod profiles constitute a reasonable upper bound for estimates of system degradation effects in terms of  $\Delta\phi_{\text{rms}}$  and  $\lambda d$ .

## 2. ANALYTIC METHODS

### 2.1 ANALYSIS

We use the thin-screen approximation to describe the signal which results after a plane wave propagates through a region of striated ionization. The derivation and validity criteria of this approximation are in reference 1. The ionization is represented by a collection of striations each of which has a profile of electron density

$$n_e(\vec{x}) = n_i(\vec{x} - \vec{x}_i, a_i)$$

where  $\vec{x}_i = (x_i, z_i)$  is the two dimensional position of the  $i$ 'th striation and  $a_i$  is its characteristic size. The electron density is uniform in the third dimension.

For a plane wave propagating in the  $z$  direction, the thin-screen approximation describes the effect of each striation on the signal to be a phase change

$$\phi_i(x - x_i, a_i) = \int n_i(\vec{x} - \vec{x}_i, a_i) dz \frac{k_0}{2n_c}$$

where  $k_0$  is the wave number of the signal  $= 2\pi/\lambda$  where  $\lambda$  is the wavelength and  $n_c = k_0^2/4\pi r_e$ , the critical electron density where  $r_e = 2.818 \cdot 10^{-13}$  cm, the electron classical radius. The total effect on the signal is then a net phase change due to all the striations of

$$\phi(x) = \sum_i \phi_i(x - x_i, a_i) .$$

We consider two extremes of electron density profile:

$$\text{The Gaussian, } n_i^g = n_0 e^{-(\vec{x} - \vec{x}_i)^2 / a_i^2}$$

which produces the phase

$$\phi_i^g = C \sqrt{\pi} a_i e^{-(x - x_i)^2 / a_i^2};$$

$$\begin{aligned} \text{the rod, } n_i^r &= n_0 \quad |\vec{x} - \vec{x}_i| < a_i \\ &= 0 \quad |\vec{x} - \vec{x}_i| > a_i \end{aligned}$$

which produces the phase

$$\phi_i^r = C 2a_i \sqrt{1 - \frac{(x - x_i)^2}{a_i^2}}$$

$$\text{where } C = \frac{k_0 n_0}{2n_c} = r_e \lambda n_0.$$

The actual profile of a striation should fall between these two extremes.

The total phase has the Fourier transform

$$\begin{aligned} F(v) &= \int dx e^{-i2\pi vx} \phi(x) \\ &= \sum_i e^{-i2\pi vx_i} F_i(v) \end{aligned}$$

$$\text{where } F_i^g(v) = C \pi a_i^2 e^{-\pi^2 v^2 a_i^2} \text{ and } F_i^r(v) = C \frac{a_i}{v} J_1(2\pi v a_i),$$

where  $J_1$  is a Bessel function.

Notice that  $F_i^g(0) = F_i^a(0) = C\pi a_i^2$  because each profile has the same electron content,  $\int dx \int dz n_i(\vec{x} - \vec{x}_i, a_i) = \pi n_0 a_i^2$ .

For a given set of striations, the phase has a power spectrum

$$P(v) = |F(v)|^2 = \sum_i F_i^2(v) + 2 \sum_{i < j} F_i(v) F_j(v) \cos[2\pi v(x_i - x_j)] .$$

The first term is independent of the positions of the striations. The second term would be larger than the first by about a factor of the total number of the striations if the cosine terms all added coherently. The discussion of the propagation effects of a collection of striations should ideally not be dependent on their particular positioning unless it can be shown that characteristics of their positioning can be predicted by the phenomenology. Since no positioning characteristics have yet come from phenomenology, the second term can be eliminated by considering the striations to be randomly placed and averaging over an ensemble of random placements. If the total area occupied by the striations is small compared to the area they are placed in, one can approximate<sup>2</sup> the probability of any two striations' placement as independent and equally likely within the area. The position,  $x_i$ , is then equally likely on a line,  $L$ , which produces an average power spectrum of

$$\overline{P(v)} = \sum_i F_i^2(v) + 2S^2 \sum_{i < j} F_i(v) F_j(v)$$

where  $S = \frac{\sin(\pi v L)}{\pi v L}$ .

By evaluating the spectrum at values of

$$v_n = \frac{n}{L} \quad n = 1, 2, 3, \dots$$

where  $S = 0$ , we are correctly ignoring the effects of the finite interval,  $L$ , on the Fourier transform. We are expanding  $\phi(x)$  in a Fourier series.

The propagation calculations will be performed for one set of random positions of the striations. This is the situation an actual radio signal encounters. We illustrate in Figures 1 and 2 the power spectrum for the one set and the average power spectrum for the Gaussian and rod profiles. The scale is normalized ( $C = 1$ ). We use a set of positions and radii furnished by Dr. W. Chesnut<sup>3</sup>(SRI). This is his "Type B" distribution consisting of 210 striations whose positions are randomly distributed over a line  $L = 64$  km. This distribution was also used in reference 1.

It is obvious that the large differences between the specific set of positions and the average at low- $\nu$  is a chance alignment in phases for the positions chosen. It appears for both Gaussian and rod spectra.

The rod spectra have a higher frequency content of course. The spectra appear to change from exponential to power law at  $\nu = 1$ . Since there are no striations in the set with  $a_i < 0.323$  km, all Bessel functions would have arguments greater than 2 for  $\nu = 1$ . They then decrease with an envelope of  $\nu^{-3/2}$ , giving the power spectrum a  $\nu^{-3}$  fall off. The Gaussian spectrum is exponential until  $\nu = 1.5$  where the Gaussian fall off should take over

The propagation calculations we will perform with the specific set of striations depend on the Fourier transform of  $e^{i\phi(x)}$ . We cannot yet analytically determine the effect of the large low- $\nu$  variation in the power spectrum of  $\phi(x)$  for the specific set on our calculated effects. Our intuition says the low- $\nu$  portion of the spectrum is unimportant so long as it is finite. We also feel that all statistical properties of the signal of interest will be unaffected by the noise-like difference between the specific and average spectrum.

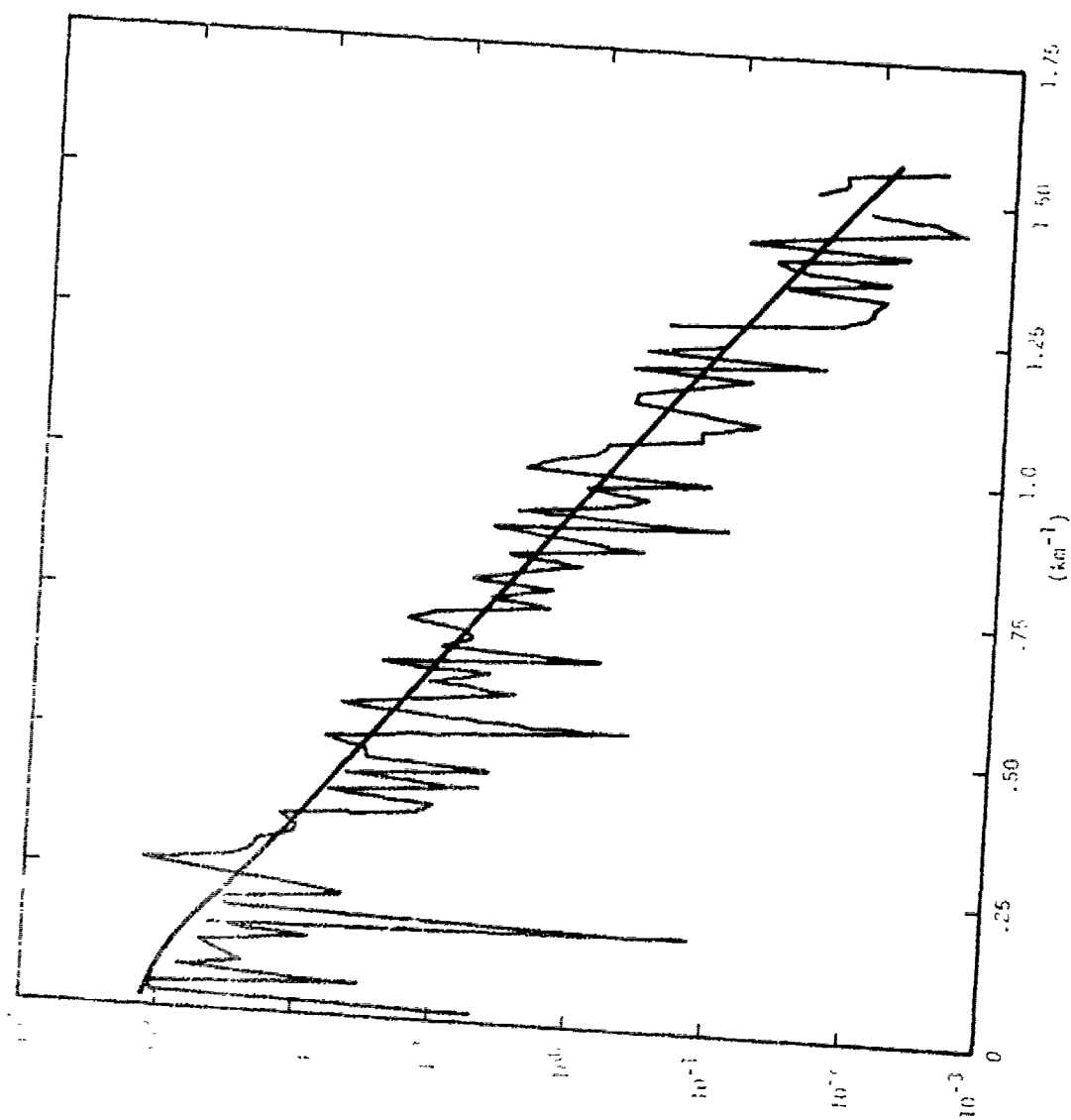


FIGURE 1 - Specific and Average Power Spectra Gaussian Profile

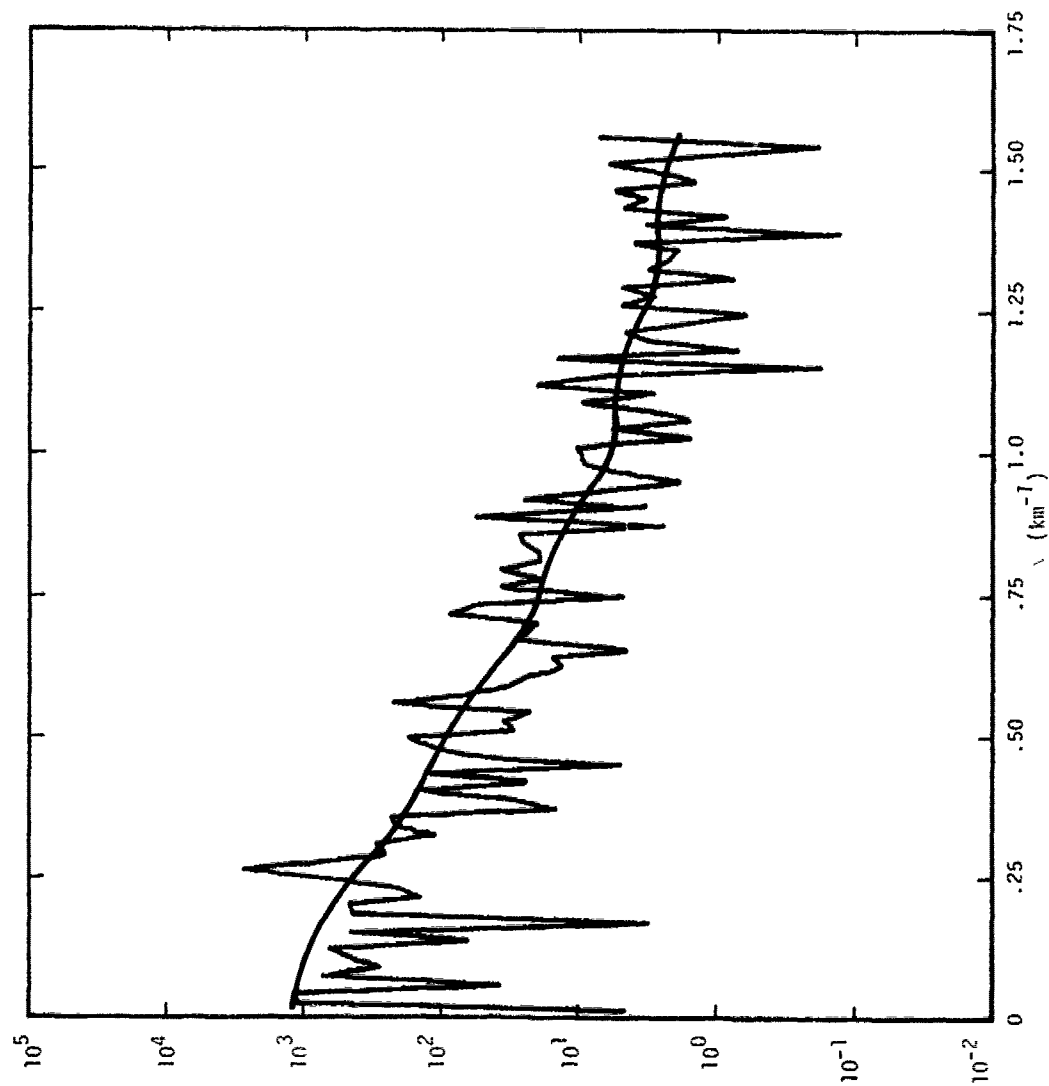


Figure 2 - Specific and Average Power Spectra Rod Profile

Consider the following statistical functions:

$$\bar{\phi} \equiv \frac{1}{L} \int_0^L dx \phi(x) = \frac{F(0)}{L}$$

$$\begin{aligned} \Delta\phi = \phi - \bar{\phi} &= \frac{1}{L} \sum_{n=-\infty}^{\infty} F\left(\frac{n}{L}\right) e^{i2\pi nx/L} - \frac{F(0)}{L} \\ &= \frac{1}{L} \sum_{n=1}^{\infty} \left[ F\left(\frac{n}{L}\right) e^{i2\pi nx/L} + F\left(\frac{-n}{L}\right) e^{-i2\pi nx/L} \right] \end{aligned}$$

That is, the Fourier transform of  $\Delta\phi$  is the same as the Fourier transform of  $\phi$  with the  $v = 0$  term removed. The auto correlation function

$$\begin{aligned} \overline{\Delta\phi(x)\Delta\phi(x+\Delta x)} &\equiv \frac{1}{L} \int_0^L dx \Delta\phi(x)\Delta\phi(x+\Delta x) \\ &= \frac{1}{L^2} \sum_{n=1}^{\infty} P\left(\frac{n}{L}\right) \left[ e^{i2\pi n\Delta x/L} + e^{-i2\pi n\Delta x/L} \right] \end{aligned}$$

which follows by substitution and the properties

$$\begin{aligned} \frac{1}{L} \int_0^L dx e^{i2\pi x/L(n+n')} &= 0 \quad n \neq -n' \\ &= 1 \quad n = -n' \end{aligned}$$

$$F\left(\frac{-n}{L}\right) = F^*\left(\frac{n}{L}\right).$$

We have the well known result that the power spectrum is the Fourier transform of the auto correlation function. In this case we are referring to power per unit length since we are interested in  $\Delta\phi$  over the interval  $(0,L)$  only. Over this interval the auto correlation function is meaningful out to  $|\Delta x| = L/2$

since beyond this value the periodicity of  $\Delta\phi$  causes it to mirror its behavior back to its value at  $\Delta x = 0$ . That is,  $\overline{\Delta\phi(x)\Delta\phi(x+L-\Delta x)} = \overline{\Delta\phi(x)\Delta\phi(x+\Delta x)}$ .

Indeed, for cases of interest to be treated later,  $\overline{\Delta\phi^2} = \frac{2}{L^2} \sum_n P(\frac{n}{L}) \gg 1$ . This causes most of the interest in the auto correlation function to be in its behavior at small  $\Delta x$ . For example, one item of interest is that value of  $\Delta x$  where

$$\overline{\Delta\phi\Delta\phi(\Delta x)} - \overline{\Delta\phi^2} = -1 \quad \text{or}$$

$$\frac{2}{L^2} \sum_n P(\frac{n}{L}) [1 - \cos(2\pi \frac{n\Delta x}{L})] = 1 \quad \text{or}$$

$$\frac{4}{L^2} \sum_n P(\frac{n}{L}) \sin^2(\frac{n\Delta x}{L}) = 1 \quad .$$

Since this will occur at a value of  $\Delta x \ll L$ , the low  $v$  (small  $n$ ) difference between the specific and average spectrum should not impact the result. Similarly for  $\pi\Delta x \ll L$ , the  $\sin^2$  weighting will vary slowly over a range of  $n$  values which smoothes out the differences between the specific and average spectrum.

The discrete distribution of  $a_i$  that we use approximates the continuous Chesnut distribution,

$$m(a)da = \frac{8M}{3\sqrt{\pi}} \frac{a_0^5}{a^6} e^{-(a_0^2/a^2)} da \quad ,$$

the number of striations of size  $a$  in an interval  $da$ . The total number of striations,  $M = 210$ .

Analytically,

$$\begin{aligned} P(v) &= \int_0^{\infty} \ln(a) da \left( C \pi a^2 e^{-\pi^2 v^2 a^2} \right)^2 \\ &= \frac{4}{3} C^2 \pi^2 a_0^4 M e^{-\sqrt{8} \pi v a_0} \end{aligned}$$

for the Gaussian profile.

Figure 1 shows the exponential behavior quite well except near  $v = 0$  where the cutoff of large striations in the discrete distribution returns the vanishing slope at the origin. From Figure 1, a measured slope of 7.83 produces the value  $a_0 = 0.881$  km.

We are interested in relating the propagation effects to a characteristic size of the distribution used and to the parameter

$$\overline{\Delta \phi^2} = \frac{2}{L^2} \sum_n P\left(\frac{n}{L}\right) = \frac{2}{L} \int_0^{\infty} P(v) dv = \frac{\sqrt{8}}{3} C^2 \pi a_0^3 \left(\frac{M}{L}\right)$$

for the continuous distribution. We have measured this quantity directly from the discrete distribution and find

$$\Delta \phi_{rms} = \sqrt{\overline{\Delta \phi^2}} = 2.0711 C .$$

Substitution of this value into the above equation produces  $a_0 = 0.761$  km. Thus there is a 10% discrepancy in determining  $a_0$  from specific and average discrete distributions using formulas for a continuous distribution. Since the geometric structure of the striations we use is fixed, we will relate the propagation effects to  $\Delta \phi_{rms}$  by variations of the parameter  $C$ .

We now discuss the signal at a distance  $d$  beyond the striated region.  
If the signal is defined as

$$E = U(x,z)e^{i(k_0 z - \omega t)}$$

and  $U(x,0) = e^{i\phi(x)}$  at the exit plane of the striations, the signal at some point  $(x_0, d)$  beyond the exit plane is obtained from the free-space parabolic equation

$$\frac{\partial U}{\partial z} + \frac{1}{2ik_0} \frac{\partial^2 U}{\partial x^2} = 0.$$

Fourier analysis of the parabolic equation shows that for

$$U(x,0) = \sum_k \hat{U}(k) e^{ikx}$$

$$U(x_0, d) = \sum_k \hat{U}(k) e^{i[kx_0 - \frac{k^2 d}{2k_0}]}$$

This method has had widespread use by workers in striation propagation and is efficient for production runs of  $U(x_0, d)$  because of fast Fourier transform techniques. This technique decomposes the signal at  $z = 0$  into plane waves, propagates each plane wave to the receiver at  $(x_0, d)$  and sums the waves to obtain  $U(x_0, d)$ . Since  $U(x,0)$  was defined only for  $0 < x < L$ , the use of a sum (Fourier series) rather than an integral specifies  $U(x,0)$  as periodic in  $L$ . We can accept this as an unimportant consequence so long as we do not evaluate the signal at values of  $d$  so far from the screen that the signal is influenced by values of  $U(x,0)$  over an interval greater than  $L$ .

The Fresnel distance for an interval  $L$  is

$$d_f = k_0 L^2 / 2 .$$

The smallest wave number of interest is  $\sim 5 \cdot 10^3 \text{ km}^{-1}$  so we need  $d < d_f \approx 10^7 \text{ km}$  for  $L = 64 \text{ km}$ , which is well satisfied for cases of interest. A further requirement when large phase variations occur is  $L/2d > \theta_M$  where  $\theta_M$  is the largest angle of a significantly contributing plane wave.

The parabolic equation is a small angle approximation to the wave equation. For each plane wave, the parameter  $k$  is  $k_0 \sin \theta_k \approx k_0 \theta_k$ . The amplitude of the plane wave is the Fourier integral

$$\hat{U}(k) = \frac{1}{L} \int_0^L e^{i[\phi(x) - kx]} dx$$

As in a previous analysis<sup>1</sup>, we may define a local phase-front angle

$$\theta(x) = \frac{1}{k_0} \frac{\partial \phi}{\partial x}$$

and the integral is

$$\hat{U}(k) = \frac{1}{L} \int_0^L e^{ik_0 \int_0^x (\phi(x') - \theta_k) dx'} dx$$

If instead of fast Fourier techniques, we evaluated the integral by stationary phase<sup>1</sup>, the points of stationary phase would be wherever

$$\theta(x_s) = \theta_k$$

and

$$\hat{U}(k) = \sqrt{\frac{2\pi i}{L_0^2 k_0}} \sum_s \frac{e^{i[\phi_0(x_s) - kx_s]}}{\sqrt{\theta'(x_s)}}$$

with a criterion for the validity of this method of  $|\theta''(x_s)|^2 \ll k_0 |\theta'(x_s)|^3$ .

Although this method of evaluation of the integral becomes inaccurate where  $\theta'(x)$  is small, it is clear that there are negligible contributions to the integral for  $\theta_k > \theta(x)_{\max}$  since there would then be no stationary points.

For a Gaussian profile

$$\theta_i(x) = \frac{-2C\pi}{k_0 a_i} (x-x_i) e^{-(x-x_i)^2/a_i^2}$$

with maximum value

$$\theta_{i_{\max}} = \sqrt{\frac{2\pi}{e}} \frac{C}{k_0} = \sqrt{\frac{\pi}{2e}} \frac{n_0}{n_c} \ll 1$$

and depending on the number of striations along the path there is some finite limit of  $\theta_k$  for which the requirement

$$d < \frac{L}{2\theta_M}$$

can be tested.

For the case of a rod profile

$$\theta_i(x) = \frac{-C}{k_0 a_i} \frac{(x-x_i)}{\sqrt{1 - \frac{(x-x_i)^2}{a_i^2}}}$$

which approaches infinity as  $|(x-x_i)|$  approaches  $a_i$  and there is no limit on  $\theta(x)$ . Indeed there are two infinities per striation in  $\theta(x)$ .

However

$$\theta'_i(x) = \frac{-C}{k_0 a_i} \frac{[(x-x_i)^2/a_i^2 + 1]}{\sqrt{1 - \frac{(x-x_i)^2}{a_i^2}}}$$

also becomes infinite so the amplitude of the plane wave contribution goes to zero.

The rod profile is an extreme idealization of the sharpness of the edge of a striation. The fact that we will evaluate  $\hat{U}(k)$  by a fast Fourier transform rather than analytically, means we are automatically smoothing the edges of the striation since the largest value of  $\theta$  obtainable is

$$\frac{\phi_i(x = x_i + a_i - \Delta x) - \phi_i(x = x_i + a_i)}{k_0 \Delta x}$$

So  $\theta_{i \max} \approx \frac{n_c}{n} \sqrt{\frac{2a_i}{\Delta x}}$  where  $\Delta x$  is the step size =  $L/N$  where  $N = 2^m$  is the number of Fourier components (plane waves) to be determined.

We have carried out the analysis for  $m = 14$  and  $15$  to see the impact on propagation effects of changing the step size and hence the maximum plane wave angle. The differences were negligible, indicating that for the cases we have treated, the largest angle of a significantly contributing plane wave was less than the largest angle obtainable by using  $m = 15$ . We do not know what this angle is for the rods. We estimate the angle for the Gaussian profile using previous analysis<sup>1</sup> which shows

$$\overline{\theta^2} = \frac{1}{(k_0 a_0)^2} \overline{\Delta\phi^2}$$

If we choose the restriction

$$d < \frac{L}{2\sqrt{\overline{\theta^2}}} = \frac{k_0 a_0 L}{2\Delta\phi_{\text{rms}}}$$

or

$$\lambda d < \frac{\pi a_0 L}{\Delta\phi_{\text{rms}}} \approx \frac{150}{\Delta\phi_{\text{rms}}}$$

we should be confident that the finiteness of the interval,  $L$ , will not impact the calculations. It turns out that the results of importance for this study occur well before this limit is reached.

The parameters  $\lambda d$  and  $\Delta\phi_{\text{rms}}$  (or  $C$ ) uniquely determine the propagation effects for a given striation structure. This can be seen from an alternate expression for the signal previously obtained<sup>1</sup>

$$U(x_0, d) = \sqrt{\frac{1}{i\lambda d}} \int dx e^{i[\Delta\phi(x) + \pi(x-x_0)^2/\lambda d]}$$

For a fixed structure,  $\Delta\phi(x)$  and  $\Delta\phi_{\text{rms}}$  are both linear in  $C$  and therefore scale together. The only other parameter in the integral is  $\lambda d$ . We may therefore do all calculations at one frequency varying  $C$  and  $d$ , present the results in terms of  $\lambda d$  and  $\Delta\phi_{\text{rms}}$  and obtain results at other frequencies by scaling.

## 2.2 IMPLEMENTATION

We choose a range of  $\lambda d$  of interest by considering the frequency range of interest to be 300 MHz to 30 GHz and the distance range of interest to be  $10^2$  to  $10^4$  km. This limits

$$10^{-3} \leq \lambda d \leq 10 \text{ km}^2.$$

We choose  $\sqrt{0.1} \leq \Delta\phi_{\text{rms}} \leq 100$  radians. Effects are negligible at and below the lower limit. The higher limit is chosen so as to insure adequate sampling of  $e^{i\phi(x)}$  by the fast Fourier transform. These limits are sufficient to demonstrate results. The transparency of the medium is assured for high altitude plasma by the restriction  $\Delta\phi_{\text{rms}} \leq 100$  radians. For a temperature around  $1000^\circ\text{K}$ , the ion-electron momentum transfer collision frequency

$$\nu_{ei}(\text{s}^{-1}) \approx 10^{-3} n_e(\text{cm}^{-3}).$$

A wave traveling a thickness  $z$  of the plasma then suffers  $\int_0^z dz 1.28 \cdot 10^{-11} n_e^2 \lambda^2$  dB of attenuation where  $\lambda$  is the wave length in km. For the 64 km width of the striations the average attenuation is

$$\bar{A} = \frac{1.28 \cdot 10^{-11} \lambda^2}{64} \int_0^L dx \int_0^z dz \sum_i n_i^2(x, z)$$

Now

$$\int_0^L dx \int_0^z dz n_0^2 e^{-2(\bar{x} - \bar{x}_i)^2 / a_i^2} = \frac{\pi n_0^2 a_i^2}{2}$$

for Gaussian profiles and

$$\int_0^L dx \int_0^z dz \begin{cases} n_0^2, & r < a_i \\ 0, & r > a_i \end{cases} = \pi n_0^2 a_i^2$$

for rods.

Therefore, the rods produce twice the average attenuation that Gaussian profiles produce for the same electron content. For Gaussian profiles we have

$$\bar{A} = 10^{-13} \lambda^2 n_0^2 \sum_i a_i^2$$

For the particular set of 210 striations we use

$$\sum_i a_i^2 = 112.62$$

and  $\bar{A} = 3.54 \cdot 10^{-11} \lambda^2 n_0^2$  dB where  $\lambda$  is in km and  $n_0$  is  $\text{cm}^{-3}$ . Similarly, we have

$$\Delta\phi_{\text{rms}} = 2.818 \cdot 10^{-3} \lambda n_0 Q_{\text{rms}}$$

where  $Q(x) = \int_0^L dz \frac{n_e(x,z)}{n_0}$  is the normalized integrated electron density in km. For the particular set used,  $Q_{\text{rms}} = 2.071$  km so  $\Delta\phi_{\text{rms}} = 5.836 \cdot 10^{-3} \cdot n_0 = 98\sqrt{\bar{A}}$ .

For  $\Delta\phi_{\text{rms}} \leq 100$ ,  $\bar{A} \leq 10^{-2}$  dB. For the rods,  $Q_{\text{rms}} = 2.55$  km, so  $\Delta\phi_{\text{rms}} = 7.186 \cdot 10^{-3} \lambda n_0$  but  $\bar{A} = 7.08 \cdot 10^{-11} \lambda^2 n_0^2 = 1.37 \cdot 10^{-6} \frac{\Delta\phi_{\text{rms}}^2}{\lambda^2}$  and absorption is negligible for  $\Delta\phi_{\text{rms}} \leq 100$ . Our procedure is to obtain  $Q(x)$  directly from the 210 striations in steps of  $64 \text{ km}/2^{15} = 1.953\text{m}$  and store it. For each value of  $\Delta\phi_{\text{rms}}$  desired we multiply  $Q(x)$  by  $\Delta\phi_{\text{rms}}/Q_{\text{rms}}$  to obtain  $\phi(x)$  and

fast Fourier transform  $e^{i\phi(x)}$  to obtain  $U(k)$ . We use the Berkeley computer library routine CFFT. Their write-up describes the inverse transform as

$$U(x,0) = \sum_{j=0}^{N-1} \hat{U}(j) e^{ik_j x}$$

where

$$k_j = \frac{2\pi j}{L}.$$

This reproduces the function  $U(x,0)$  exactly at the sampled points  $x_p = \frac{Lp}{N}$   $p = 0,1,2,\dots,N-1$ . Unfortunately, it is not the appropriate inverse transform. The proper one for best fit to the function at all  $x$  is

$$U(x,0) = \sum_{j=0}^{N/2} \hat{U}(j) e^{ik_j x} + \sum_{j=1}^{N/2-1} \hat{U}(N-j) e^{-ik_j x}$$

which also reproduces the function exactly.

This is the proper truncation of the infinite Fourier series which has both positive and negative wave numbers. The one described in the Berkeley write-up replaced the negative wave numbers by wave numbers one period higher which introduces high frequency (wave number) content the original function did not have.

Having the  $\hat{U}(j)$  for the specified  $\Delta\phi_{rms}$ , we then modify  $U(j)$  for each of a set of values of  $\lambda d$  to describe the wave a distance  $d$  away as

$$\hat{U}(j,d) = \hat{U}(j) e^{-ik_j^2 d \lambda / 4\pi} \quad j = 0, 1, \dots, N/2 ;$$

$$\hat{U}(N-j,d) = \hat{U}(N-j) e^{-ik_j^2 \lambda d / 4\pi} \quad j = 1, 2, \dots, N/2-1 .$$

and call the inverse CFFT to obtain  $U(x,d)$ . The routine performs the inverse transform properly.

For the rod profiles we redid the calculations using both  $2^{14}$  and  $2^{15}$  Fourier components and found the results consistent, signifying the sampling of  $Q(x)$  to be adequate.

### 3. RESULTS

In this section we compare the propagation effects of rod profile striations and Gaussian striations. Since the computer routine calculates the signal at  $2^{15}$  values of  $x$  over a 64 km range, we have a good sampling for statistics. The output of the routine includes the mean intensity,  $\overline{|U|^2}$ , which should be unity by conservation of energy (this check on the results was unity for all cases), and the scintillation index,

$$S_4 \equiv \sqrt{|U|^4 - 1}.$$

The distribution of the intensity is obtained by printing the number of samples in bins 3 dB wide. The following table has the percent of samples in each bin for those cases where at least 0.5% of the samples were less than -3 dB.

The columns begin with the percent less than -30 dB and work up in 3 dB increments with the last column for samples  $\geq 12$  dB. The T signifies a finite percentage less than or equal to .05. The calculations were carried out for  $\lambda d$  up to the lesser of 10 and  $100/\Delta\phi_{rms}$ . We chose to stop the tabulation at  $\lambda d$  the lesser of 10 and  $10/\Delta\phi_{rms}$ .

The results for Gaussian profiles do not enter the table for  $\lambda d < (\Delta\phi_{rms})^{-1}$ . The results for rod profiles do not enter the table for  $\lambda d < 10^{-1/2}(\Delta\phi_{rms})^{-2}$  within the range of  $\lambda d$  of interest.

If one considers the question of how one scales a critical quantity with wave length, one will get different scaling for the rod and Gaussian profiles. For example, consider the value of on-axis electron density,  $n_0$ , below which more than 99.5% of the signal is above -3 dB. For the Gaussian

| $\log_{10} \frac{1}{\sigma_{rms}}$ |   | $\log_{10} d$ | <-30 | -27 | -24 | -21 | -18 | -15 | -12 | -9  | -6   | -3   | 0    | 3    | 6    | 9   | 12  | >12 |
|------------------------------------|---|---------------|------|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|-----|-----|-----|
| r                                  | g | -5            |      |     |     |     |     |     |     |     | 0    | 3.3  | 52.5 | 43.4 | 0.8  | 0   |     |     |
|                                    |   |               |      |     |     |     |     |     |     | 0   | 1.9  | 52.6 | 45.0 | 0.5  | 0    |     |     |     |
| r                                  | g | 1             |      |     |     |     |     |     |     | 0   | 1.6  | 8.8  | 40.5 | 48.7 | 0.4  | 0   |     |     |
|                                    |   |               |      |     |     |     |     |     |     | 0   | 0.7  | 7.2  | 39.3 | 52.9 | 0    |     |     |     |
| r                                  | g | 0             |      |     |     |     |     |     |     | 0   | 0.2  | 5.3  | 51.5 | 41.6 | 1.3  | 0   |     |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | 0             | T    | T   | T   | T   | T   | 0.1 | 0.4 | 1.2 | 3.4  | 14.5 | 39.8 | 32.1 | 8.0  | 0.2 | 0   |     |
|                                    |   |               |      |     |     |     |     |     |     | 0   | 0.9  | 9.9  | 48.3 | 37.2 | 3.7  | T   | 0   |     |
| r                                  | g | .5            | T    | T   | 0.1 | 0.2 | 0.3 | 1.0 | 1.5 | 5.0 | 11.1 | 16.2 | 23.6 | 27.9 | 12.9 | 0.3 | 0   |     |
|                                    |   |               | 0.1  | 0.1 | 0.1 | 0.1 | 0.3 | 0.8 | 1.5 | 1.9 | 9.0  | 21.1 | 23.7 | 30.2 | 12.0 | 0   |     |     |
| r                                  | g | 1             | T    | T   | 0.1 | 0.4 | 0.6 | 1.2 | 2.6 | 4.0 | 7.8  | 13.1 | 29.2 | 28.8 | 11.9 | 0.3 | 0   |     |
|                                    |   |               | T    | T   | T   | 0.5 | 1.7 | 2.2 | 2.2 | 3.3 | 6.8  | 13.3 | 26.8 | 33.2 | 9.8  | 0.3 | 0   |     |
| r                                  | g | .5            |      |     |     |     |     | 0   | T   | 0.1 | 3.0  | 55.2 | 40.2 | 1.5  | 0    |     |     |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -1            | T    | T   | T   | T   | T   | 0.1 | 0.9 | 3.0 | 13.3 | 43.1 | 32.5 | 6.7  | 0.2  | 0   |     |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -1.5          | T    | T   | 0.1 | 0.2 | 0.6 | 0.9 | 1.8 | 3.6 | 8.6  | 17.3 | 28.6 | 26.4 | 10.8 | 1.2 | 0   |     |
|                                    |   |               |      |     |     |     |     | T   | 0.1 | 0.2 | 0.5  | 13.9 | 52.1 | 24.3 | 8.1  | 0.8 | 0   |     |
| r                                  | g | 0             | 0.1  | 0.1 | 0.2 | 0.3 | 0.8 | 1.7 | 2.7 | 4.0 | 9.3  | 17.0 | 24.5 | 27.5 | 10.4 | 1.5 | 0   |     |
|                                    |   |               | T    | T   | 0.1 | 0.2 | 0.6 | 1.1 | 1.8 | 5.0 | 13.0 | 18.9 | 23.0 | 22.4 | 11.5 | 2.5 | 0   |     |
| r                                  | g | .5            | 0.1  | 0.1 | 0.2 | 0.3 | 0.6 | 1.4 | 2.4 | 5.3 | 8.9  | 16.5 | 25.2 | 27.2 | 10.5 | 1.2 | 0   |     |
|                                    |   |               | 0.1  | 0.1 | 0.2 | 0.5 | 0.8 | 1.7 | 3.0 | 6.2 | 9.4  | 16.4 | 22.1 | 25.5 | 13.4 | 0.6 | 0   |     |
| r                                  | g | -2.5          |      |     |     | 0   | T   | T   | T   | T   | 0.2  | 2.9  | 59.2 | 36.0 | 1.6  | 0   |     |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -2            | T    | T   | T   | T   | T   | 0.1 | 0.1 | 0.4 | 1.9  | 9.7  | 49.6 | 32.9 | 5.1  | 0.1 | 0   |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -1.5          | T    | T   | 0.1 | 0.1 | 0.3 | 0.6 | 1.3 | 2.9 | 6.0  | 16.7 | 34.4 | 27.3 | 9.5  | 0.8 | 0   |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -1            | 0.1  | 0.1 | 0.2 | 0.3 | 0.7 | 1.4 | 2.7 | 5.5 | 10.3 | 18.0 | 24.3 | 23.6 | 10.8 | 2.0 | T   | 0   |
|                                    |   |               |      |     | 0   | T   | T   | T   | 0.2 | 0.3 | 13.0 | 59.1 | 20.4 | 4.5  | 2.2  | 0.2 | 0   |     |
| r                                  | g | -.5           | 0.1  | 0.1 | 0.3 | 0.5 | 0.8 | 1.7 | 3.1 | 5.6 | 10.5 | 17.1 | 23.5 | 22.5 | 11.8 | 2.2 | T   | 0   |
|                                    |   |               | T    | T   | 0.1 | 0.2 | 0.3 | 0.6 | 1.3 | 3.4 | 16.0 | 26.2 | 18.6 | 19.0 | 11.1 | 2.9 | 0.3 | 0   |
| r                                  | g | 0             | 0.1  | 0.1 | 0.2 | 0.4 | 0.9 | 1.9 | 3.4 | 5.9 | 11.1 | 16.8 | 23.1 | 22.1 | 11.9 | 2.2 | 0   |     |
|                                    |   |               | 0.1  | 0.1 | 0.2 | 0.5 | 0.7 | 1.3 | 2.9 | 5.4 | 10.2 | 18.2 | 23.7 | 23.0 | 12.2 | 1.6 | 0   |     |
| r                                  | g | 1.5           | 0    | T   | T   | T   | T   | 0.1 | 0.3 | 0.7 | 2.0  | 5.6  | 57.0 | 29.0 | 5.6  | 0.2 | 0   |     |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -2.5          | T    | T   | T   | 0.1 | 0.2 | 0.4 | 0.8 | 1.8 | 3.9  | 10.1 | 46.1 | 28.4 | 7.5  | 0.5 | T   | 0   |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -2            | 0.1  | 0.1 | 0.1 | 0.2 | 0.5 | 1.0 | 1.9 | 3.5 | 7.2  | 16.5 | 31.6 | 26.1 | 10.0 | 1.3 | T   | 0   |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -1.5          | 0.1  | 0.1 | 0.2 | 0.4 | 0.7 | 1.6 | 2.9 | 5.5 | 10.1 | 17.0 | 23.9 | 24.1 | 11.7 | 1.7 | T   | 0   |
|                                    |   |               | 0    | T   | T   | T   | T   | T   | 0.1 | 0.1 | 0.2  | 12.7 | 60.4 | 21.6 | 2.9  | 1.4 | 0.7 | 0.1 |
| r                                  | g | -1            | 0.1  | 0.1 | 0.2 | 0.4 | 0.8 | 1.6 | 3.0 | 5.6 | 10.3 | 17.1 | 23.5 | 23.6 | 11.8 | 1.6 | 0.1 | 0   |
|                                    |   |               | T    | T   | 0.1 | 0.1 | 0.3 | 0.4 | 0.9 | 2.5 | 17.8 | 30.1 | 18.4 | 15.5 | 10.3 | 3.0 | 0.4 | 0.1 |
| r                                  | g | -.5           | 0.1  | 0.1 | 0.2 | 0.3 | 0.7 | 1.6 | 3.0 | 5.9 | 10.5 | 17.3 | 23.7 | 23.2 | 11.5 | 1.9 | T   | 0   |
|                                    |   |               | 0.1  | 0.1 | 0.2 | 0.4 | 0.7 | 1.6 | 3.1 | 6.8 | 11.1 | 16.7 | 23.4 | 22.3 | 11.2 | 2.3 | 0.1 | 0   |
| r                                  | g | 2             | T    | T   | T   | 0.1 | 0.1 | 0.3 | 0.7 | 1.4 | 3.6  | 11.4 | 46.0 | 29.0 | 6.8  | 0.5 | T   | 0   |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -2.5          | T    | T   | 0.1 | 0.2 | 0.4 | 0.6 | 1.6 | 3.2 | 7.3  | 16.7 | 32.2 | 27.0 | 9.4  | 1.1 | T   | 0   |
|                                    |   |               |      |     |     |     |     |     |     |     |      |      |      |      |      |     |     |     |
| r                                  | g | -2            | 0.1  | 0.1 | 0.1 | 0.4 | 0.7 | 1.4 | 2.8 | 5.4 | 10.2 | 17.5 | 24.9 | 23.3 | 11.0 | 2.0 | 0.1 | 0   |
|                                    |   |               | 0    | T   | T   | T   | T   | T   | 0.1 | 0.1 | 0.2  | 12.5 | 60.5 | 22.0 | 3.0  | 0.8 | 0.6 | 0.2 |
| r                                  | g | -1.5          | 0.1  | 0.1 | 0.2 | 0.4 | 0.8 | 1.6 | 3.3 | 6.0 | 10.1 | 17.1 | 23.9 | 22.7 | 11.7 | 2.0 | T   | 0   |
|                                    |   |               | T    | T   | 0.1 | 0.1 | 0.2 | 0.4 | 0.9 | 1.8 | 18.8 | 32.2 | 18.6 | 18.3 | 9.2  | 3.4 | 0.8 | 0.1 |
| r                                  | g | -1            | 0.1  | 0.1 | 0.2 | 0.4 | 0.8 | 1.6 | 3.0 | 5.5 | 10.3 | 17.3 | 24.0 | 23.2 | 11.6 | 1.8 | T   | 0   |
|                                    |   |               | 0.1  | 0.1 | 0.2 | 0.4 | 0.8 | 1.7 | 3.2 | 7.8 | 11.7 | 17.3 | 22.0 | 21.2 | 10.6 | 2.6 | 0.2 | 0   |

profile,

$$\Delta\phi_{rms}\lambda d = 1 = r_e \lambda^2 n_0 d Q_{rms}$$

and  $n_0$  scales as  $\lambda^{-2}$  for a given structure and  $d$ . For the rod profiles,

$$(\Delta\phi_{rms})^2 \lambda d = 10^{-1/2} = r_e^2 \lambda^3 n_0^2 d Q_{rms}$$

and  $n_0$  scales as  $\lambda^{-3/2}$  for a given structure and  $d$ .

Therefore, the important question of the payoff of higher frequencies is bounded by the  $\lambda^{-2}$  and  $\lambda^{-3/2}$  scaling for Gaussian and rod profiles. Earlier work<sup>4</sup> by Mission Research Corporation (MRC) had suggested that propagation effects depended on  $\Delta\phi_{rms}$  alone, which means  $n_0$  scales as  $\lambda^{-1}$ . This is because their analysis assumed  $\lambda d \gg 1$ .

We illustrate this difference by including in the output the average bit error probability,  $\overline{P_e}$ , for noncoherent frequency shift key (FSK) modulation according to MRC's formula<sup>4</sup>,

$$\overline{P_e} = \int_0^\infty P(\gamma) P_e(\gamma) d\gamma$$

where  $P(\cdot)$  is the probability density that the signal to noise ratio, (S/N), is  $\gamma$  and  $P_e(\gamma) = \frac{1}{2} e^{-\gamma/2}$  is the bit error probability for a given (S/N) of  $\gamma$ . If  $\gamma_0$  is the (S/N) in the absence of the striations then we use  $\gamma = \gamma_0 |U_i|^2$  and

$$\int_0^\infty P_e(\gamma) P(\gamma) d\gamma = \frac{1}{N} \sum_{i=1}^N P_e(\gamma_0 |U_i|^2) = \overline{P_e}$$

where the sum is over the  $N = 2^{15}$  values of  $x$ . We illustrate the results in Figure 3, where the line labeled MRC is from their report. The case chosen is for  $\gamma_0 = 31.623$  (15 dB). The bracketed lines near MRC's curve show our results for  $\lambda d = 10 \text{ km}^2$ . The brackets signify the spread between rod(r) and Gaussian(g) profiles. We approach the MRC result at the high limit of  $\lambda d$ . To the right of the MRC curve are three lines signifying the results for  $\lambda d = 0.1(r)$ ,  $\lambda d = 0.1(g)$  and  $\lambda d = 0.01(g)$ . Sprinkled in between are various other points for other cases. It is apparent that at lower values of  $\lambda d$ ,  $\overline{P_e}$  gets better faster for Gaussian than rod profiles. The best way to present results of this type is on a  $\lambda d$  versus  $\Delta\phi_{\text{rms}}$  plane. We demonstrate for two contours of  $\overline{P_e} = 10^{-4}$  and  $10^{-2}$  in Figure 4. The mesh points are the cases we have run. We sketch the contours so they are on the correct side of the mesh point values.

The utility of the  $\lambda d$ - $\Delta\phi_{\text{rms}}$  chart is apparent for putting the system-effect problem in perspective. Since  $10^{2.5} \leq d \leq 10^{3.5}$ , in general we see that UHF( $\lambda=10^{-3}$ ) lies in a band at the top where trouble begins in the region  $0.3 < \Delta\phi < 1$  for both profiles. S Band( $\lambda=10^{-4}$ ) is below this band where trouble begins in the region  $1 < \Delta\phi_{\text{rms}} < 3$  for rods and  $3 < \Delta\phi_{\text{rms}} < 20$  for Gaussian profiles. Finally, K Band( $\lambda=10^{-5}$ ) is below this band with  $3 < \Delta\phi_{\text{rms}} < 10$  for rods and  $20 < \Delta\phi_{\text{rms}} < 100$  for the Gaussian profiles. Thus, the highest frequencies are the most affected by uncertainties in the profile form.

This chart was used by researchers in an attempt to put the measured scintillation index in perspective. We sketch contours of  $S_4$  for Gaussian and rod profiles in Figures 5 and 6. The Gaussian profiles give rise to large focusing effects which lead to large values of  $S_4$  in the focal region mentioned in previous work<sup>1</sup>. From the table we saw that only Gaussian profiles produced

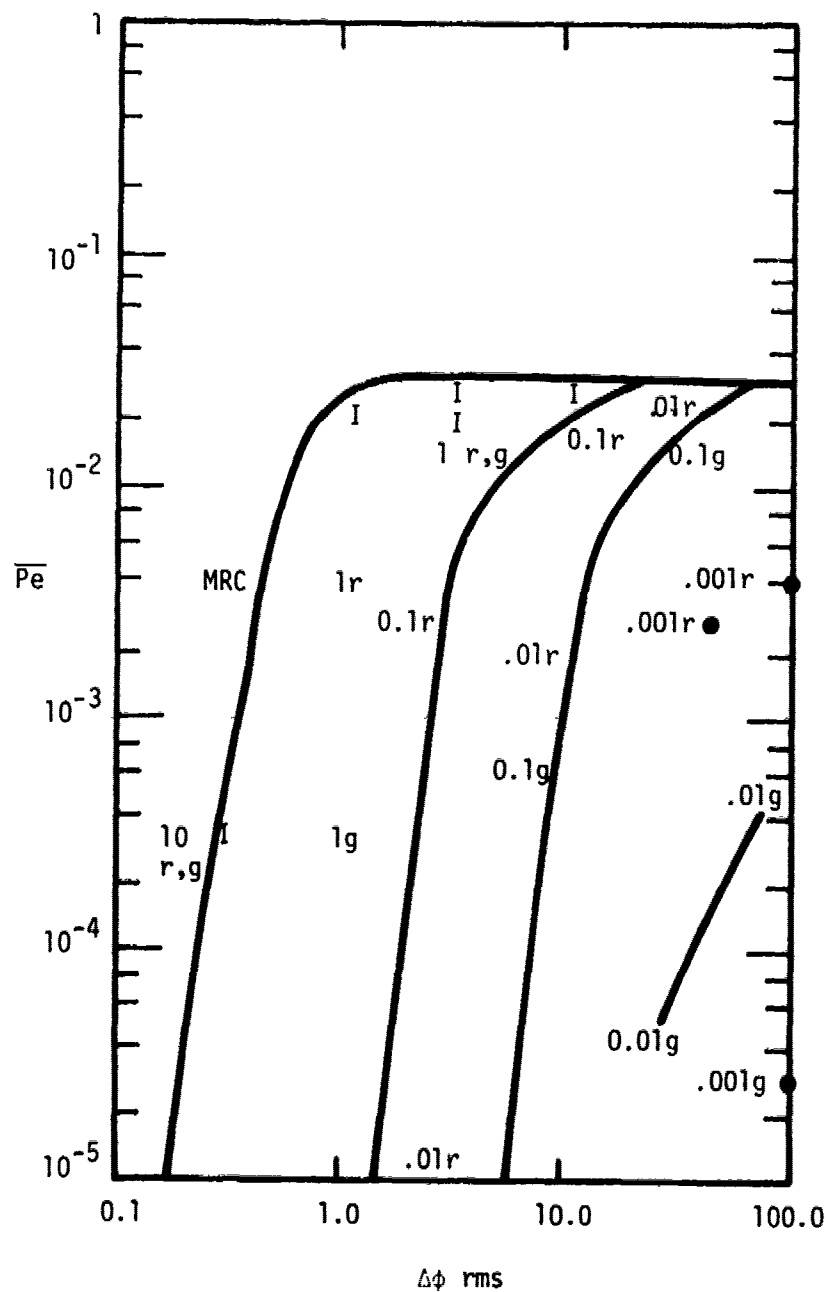


FIGURE 3 - Average Bit Error Probability Versus rms Phase Fluctuation for Noncoherent FSK Modulation and 15db Average Signal-to-Noise Ratio

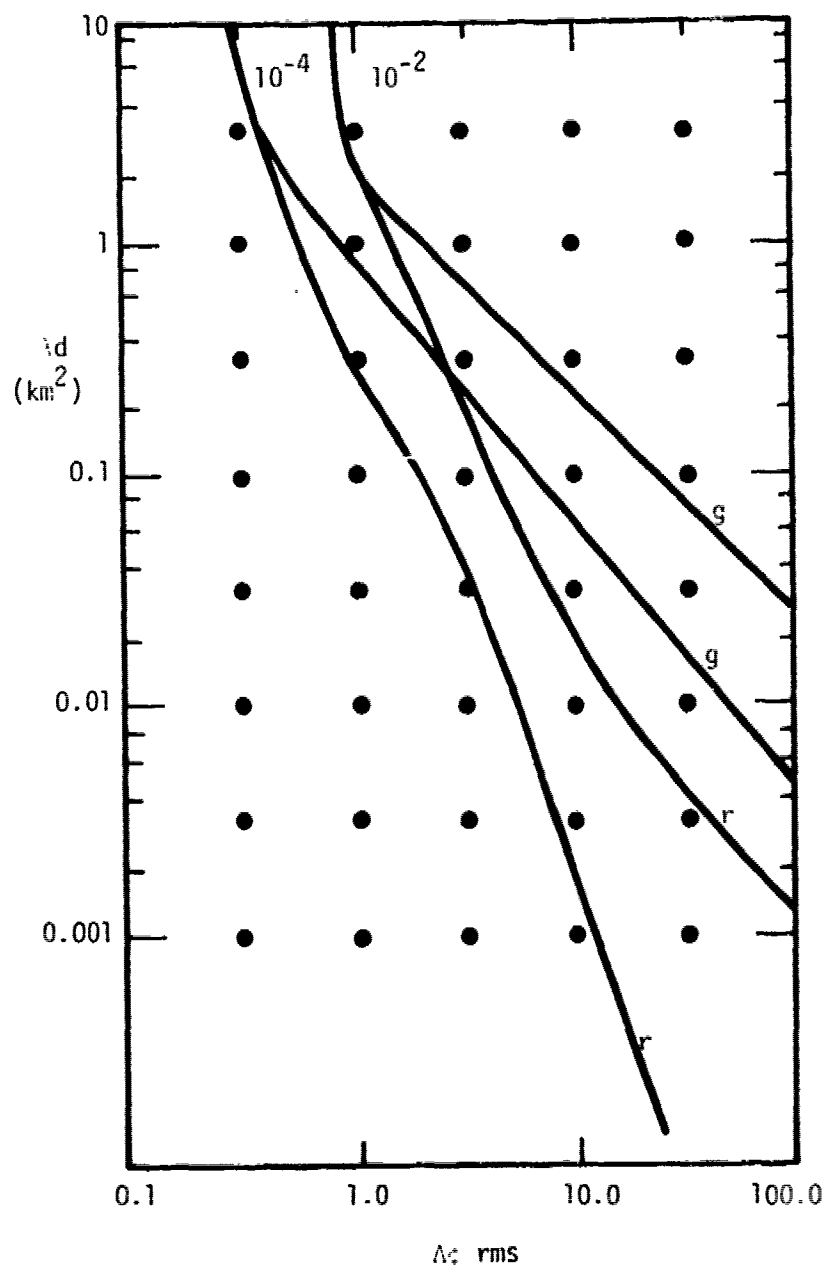


FIGURE 4 - Contours of Bit Error Probability

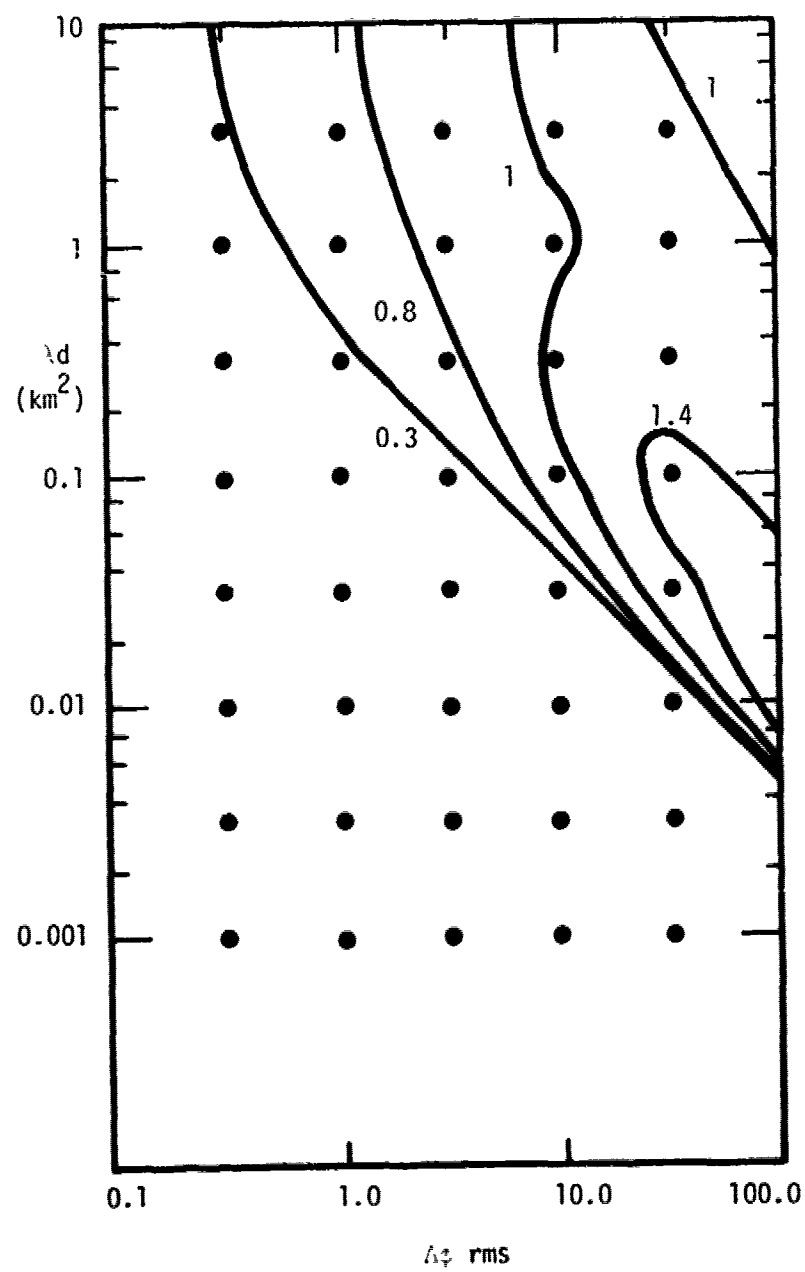


FIGURE 5 - Contours of  $S_4$ ; Gaussian

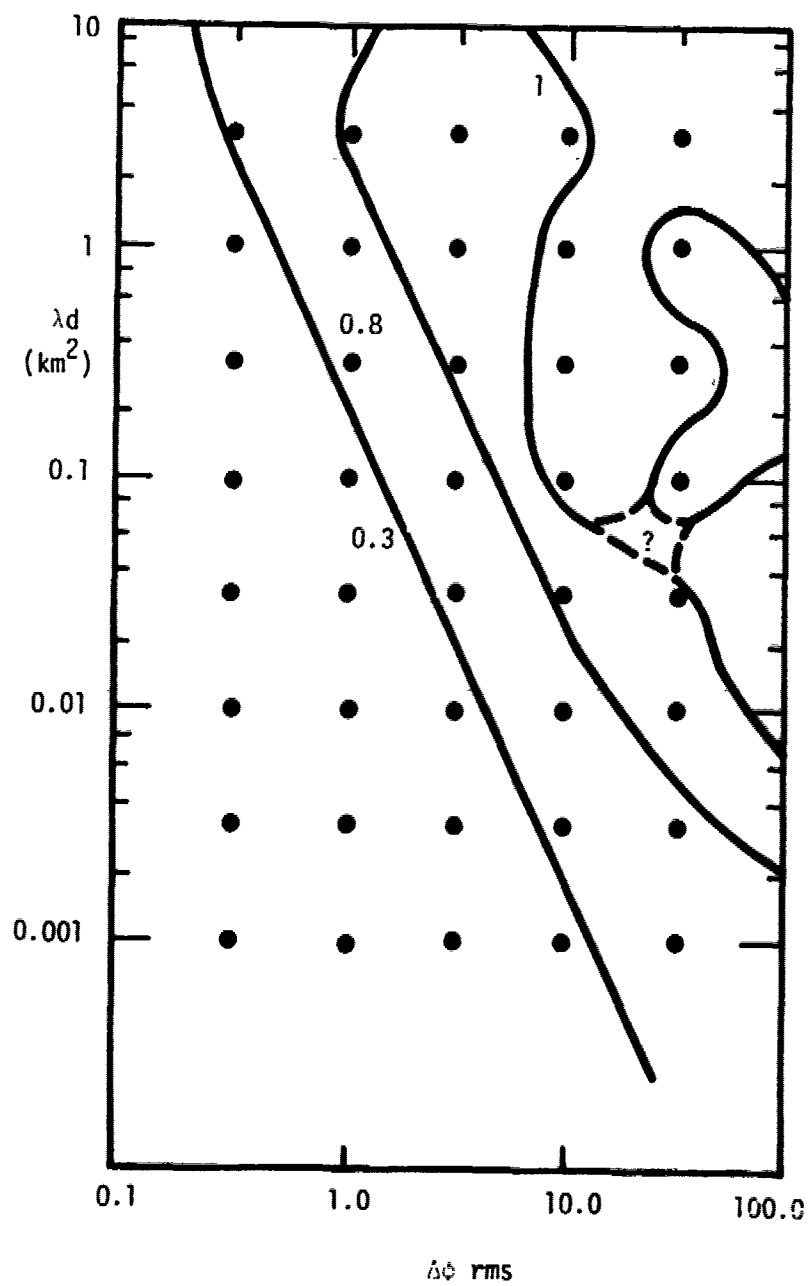


FIGURE 6 - Contours of  $S_4$ ; Rod

signals  $\geq 12$  dB. The rod profiles do not give rise to high values of  $S_4$ , and there does not seem to be any consistency as to which values are above or below 1 as can be seen by the uncertainty in drawing the  $S_4 = 1$  contour in Figure 6.

We believe  $S_4$  should be abandoned as a measure of the severity of propagation effects since it is influenced by intensity peaks which are unimportant for degradation. Comparisons of Figures 4, 5 and 6 show correlation of the onset of trouble ( $\overline{P_e} = 10^{-4}$  and  $S_4 = 0.3$ ) but we should have a better measure of the degree of trouble. As an example, we show in Figure 7 the contour for 1% probability of fades  $> 12$  dB for rod and Gaussian profiles. Here, the separate scaling is apparent. The correlation of this contour with the  $S_4 = 0.8$  contour of Figure 6 is not bad for a small range, but the correlation with  $S_4$  in Figure 5 for the Gaussian profiles is non-existent.

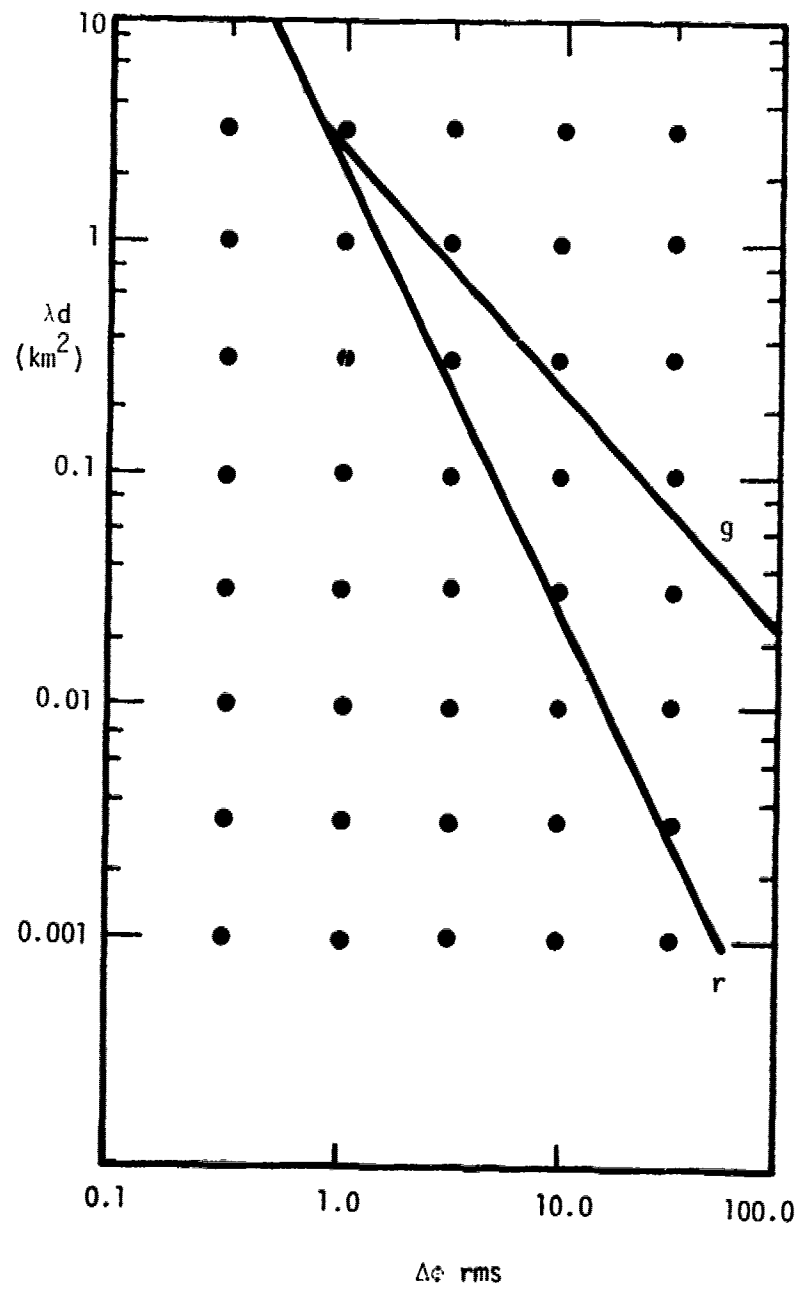


FIGURE 7 - Contours for 1% Probability of Fades > 12dB

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